Problem with a solution proposed by Arkady Alt, **San Jose** , **California**, **USA**
Compute

Find $\lim\limits_{n\to\infty} \dfrac{\left(1+\dfrac{1}{n(n+a)}\right)^{n^3}}{\left(1+\dfrac{1}{n+b}\right)^{n^2}}$.

Solution.

$$\ln \frac{\left(1+\dfrac{1}{n(n+a)}\right)^{n^3}}{\left(1+\dfrac{1}{n+b}\right)^{n^2}} = n^3 \ln\left(1+\frac{1}{n(n+a)}\right) - n^2 \ln\left(1+\frac{1}{n+b}\right) =$$

$$n^3\left(\frac{1}{n(n+a)} - \frac{1}{n^2(n+a)^2} + o\left(\frac{1}{n^4}\right)\right) - n^2\left(\frac{1}{n+b} - \frac{1}{2(n+b)^2} + o\left(\frac{1}{n^2}\right)\right) =$$

$$\frac{n^2}{n+a} - \frac{n}{(n+a)^2} - \frac{n^2}{n+b} + \frac{n^2}{2(n+b)^2} + o(1) = \frac{n^2(b-a)}{(n+a)(n+b)} + \frac{n^2}{2(n+b)^2} + o(1) \sim b-a+$$

Hence, $\lim\limits_{n\to\infty} \dfrac{\left(1+\dfrac{1}{n(n+a)}\right)^{n^3}}{\left(1+\dfrac{1}{n+b}\right)^{n^2}} = e^{b-a}\sqrt{e}$.